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TPO - REGULATOR WITH MASV METHOD

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Abstract: The paper is focused **TPO – regulator** (TPO – Time Pseudo Optimization) with classical MASV (Method Aggregate State Variables) method. Regulator uses dynamical parameters **T, D** and optimization element, which calculates so called better **T, D** in every time t. Regulation will be finished when error vector **e** is small.

Key words: Nonlinear Control, Optimization, Aggregate State Variables.

1 Introduction

Classical formulation of the MASV method does not solve the control in the finite time and construction algorithm uses the control in the infinite time. Time optimization cannot use this formulation. There are not dynamical parameters **T, D**.

In the paper we formulate **TPO - regulator** (TPO – Time Pseudo Optimization) with finite model and dynamical parameters **T, D**.

2 Mathematical model of control system

Let a mathematical model of the nominal nonlinear subsystem be considered

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},t) + \mathbf{G}(\mathbf{x},t)\mathbf{u}\,, \qquad \mathbf{x}(0) = \mathbf{x}_0 \qquad (1)$$

$$\mathbf{x} = [x_1, x_2, \ldots, x_n]^T, \quad \dim \mathbf{x} = n$$

$$\mathbf{u} = [u_1, u_2, \ldots, u_m]^T, \quad \dim \mathbf{u} = m$$

$$\mathbf{f} = [x_2, \ldots, f_{r_1}, x_{r_1+2}, \ldots, f_{r_2}, x_{r_2+2}, \ldots, f_n]^T, \quad \dim \mathbf{f} = n$$

$$\mathbf{G} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ g_{r_1 1} & g_{r_1 2} & \cdots & g_{r_1 m} \\ 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ g_{r_2 1} & g_{r_2 2} & \cdots & g_{r_2 m} \\ 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ g_{n1} & g_{n2} & \cdots & g_{nm} \end{bmatrix},$$

$$\dim \mathbf{G} = (n, m)\,,$$

$$r_j = r_{j-1} + n_j,\, r_0 = 0;\; j = 1,2,\ldots,m$$

$$n = r_m = \sum_{j=1}^{m} n_j$$

where
x - the vector of the state variables,
u - the vector of the control variables,

f - continuous vector functions,
G - the matrix of the continuous functions $g_{i,j}$,
n - the number of the state variables (the order of the nonlinear subsystem),
n_j - the partial order,
m - the number of the control variables,
T - the transpose symbol,
dim - the dimension of a vector or the order of a matrix.

The condition of controllability of the nominal nonlinear subsystem (1) must hold [Zítek & Víteček 1999]

$$\operatorname{rank}\mathbf{G}(\mathbf{x},t)=m \tag{3}$$

It is supposed that $i = 1, 2,\ldots, n$; $j = 1, 2,\ldots, m$ and strictly not distinguished between a subsystem (system) and a model in the entire the following text.

3 Control algorithms design – MASV method

The task of the optimal tracking control design is the determination of the feedback control

$$\mathbf{u}=\mathbf{u}(\mathbf{x},\mathbf{x}^{w},t) \tag{5}$$

for the controllable nominal standard nonlinear subsystem (1), which for a given state trajectory $\{\boldsymbol{x}^{w}(t)\}$ ensures its tracking by a real state trajectory $\{\boldsymbol{x}(t)\}$ so that the value of the quadratic objective functional

$$J=\int_{0}^{\infty}\left(\mathbf{e}^{T}\mathbf{D}^{T}\mathbf{D}\mathbf{e}+\dot{\mathbf{e}}^{T}\mathbf{D}^{T}\mathbf{T}^{2}\mathbf{D}\dot{\mathbf{e}}\right)dt \tag{6}$$

$$\mathbf{e}=\mathbf{x}^{w}-\mathbf{x},\ \dim\mathbf{e}=n,$$

$$\lim_{t\to\infty}\mathbf{e}(t)=\lim_{t\to\infty}\dot{\mathbf{e}}(t)=0 \tag{7}$$

is minimal,
where
e - the error vector,
D - the constant nonnegative aggregation matrix [dim **D** = (m,n), rank (**DG**) = m],
T - the diagonal matrix of positive time constants T_j of the order m, i.e.

$$\mathbf{T}=\operatorname{diag}[T_1,T_2,\ldots,T_m] \tag{8}$$

By the method of the aggregation of the state variables, it is possible to obtain the optimal feedback control (Zítek & Vítecek 1999)

$$\mathbf{u}=\left[\mathbf{D}\mathbf{G}(\mathbf{x},t)\right]^{-1}\left\{\mathbf{T}^{-1}\mathbf{D}\mathbf{e}+\mathbf{D}\left[\dot{\mathbf{x}}^{w}-\mathbf{f}(\mathbf{x},t)\right]\right\} \tag{9}$$

which causes the aggregated optimal closed-loop control system

$$\mathbf{D}\dot{\mathbf{e}}+\mathbf{T}^{-1}\mathbf{D}\mathbf{e}=0,\qquad \mathbf{e}(0)=\mathbf{e}_0 \tag{10}$$

and minimal value of the quadratic objective functional (6)

$$J^{*}=\mathbf{e}_0^{T}\mathbf{D}^{T}\mathbf{T}\mathbf{D}\mathbf{e}_0 \tag{11}$$

If the elements d_{ji} of the aggregation matrix $\boldsymbol{D}$ will be chosen in accordance with the formulas

$$\left.\begin{array}{llll} d_{ji}=0 & \text{for} & i\le r_{j-1} & \text{or}\quad i>r_j\\ d_{ji}>0 & \text{for} & r_{j-1}<i\le r_j & \\ d_{jr_j}=1 & & & \end{array}\right\} \tag{12}$$

then the characteristic polynomial of the aggregated optimal closed-loop control system (10) will be written in the form

$$N(s)=\prod_{j=1}^{m}N_j(s),$$

$$N_j(s)=\left(\frac{1}{T_j}+s\right)\sum_{p=r_{j-1}+1}^{r_j}d_{jp}s^{p-r_{j-1}-1} \tag{13}$$

where
s - the complex variable in the Laplace transform,
N_j - the characteristic polynomial of the j-th autonomous control subsystem of the partial order n_j.

It is obvious that in this case the optimal closed-loop control system consists of m

autonomous linear control subsystems whose desired dynamic behaviour can be ensured by a suitable choice of the time constants T_j and coefficients d_{ji} of their characteristic polynomials (13), i.e. by a suitable choice of the matrix $\boldsymbol{T}$ and $\boldsymbol{D}$. It is very important that the quadratic objective functional (6) has only an auxiliary purpose.

The feedback control (9) demands knowledge of the exact mathematical model of the nominal nonlinear dynamic subsystem (1) The control **u** is non-robust and is often called the equivalent control. It ensures the aggregated optimal closed-loop control system (10) from which after the completion with the equations

$$\dot{e}_i = e_{i+1}, \quad i \neq r_j \tag{14}$$

the full optimal closed-loop control system can be obtained in the form

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e}, \tag{15}$$

which has the characteristic polynomial (13).

VŠB – TUO, Department of Automatization under the leadership of prof. Ing. A. Víteček has been dealing with this problem for a long time. There were lot of papers created. [5], [6], [7], [8], [9], [10], [11], [12], [13].

4 The regulator of time pseudo optimization – TPO regulator

Solution of time optimization with the help of mathematical means is a very demanding problem. You can find the formulation of these problems in the area of MASV method in articles [1], [2], [3].

In the following text we will project the so-called TPO – regulator, which will be realized from the point of view of time optimization approximate solution of the problem.

The regulator is demonstrated on fig.1.

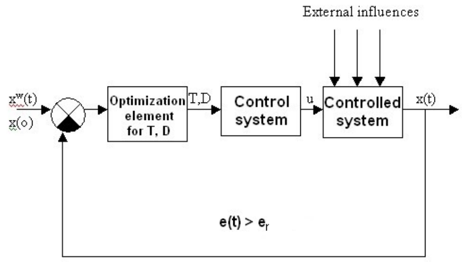

Fig.1 – TPO regulator with MASV method

Optimization element for T,D – finding the most fitting **T,D** for given time t according to given criterion.

As criterion of optimality for finding (T(t), D(t)) we can use

$$I(\mathbf{T}, \mathbf{D}) = \mathbf{e}^T(t)\mathbf{D}^T\mathbf{T}\mathbf{D}\mathbf{e}(t) \tag{16}$$

Control system – finding the value of controlled u(t) with the use of MASV method.

Controlled system – finding the value of the trajectory in time t.

5 Remarks to TPO – regulator

- **Why time?** We want to find the shortest time t_f, so that the error **x** from trajectory $\mathbf{x_w}$ was small.
- **Why pseudo optimization?** At least for two reasons:
  - we are not able to find optimal **T,D**
  - we are searching for control solution, that ends in case that we are near trajectory x_w
- **Where can be TPO - regulator used?** Certainly everywhere where the MASV method was used. We know about elementary method how to build optimization element for each problem.
- **What is the contribution of TPO – regulator?** Speedup of procuration of monitored trajectory.

6 The problems that are related with TPO - regulator

It is very important to solve following problems for the given system for setting the TPO – regulator:

- Finding the domain of convergency for **T,D**
- Finding nearly the best procedure for calculation of **T,D**
- Finding right norms for approximation of error

Basic variations of these problems are solved. In following articles we describe these problems in detail.

Conclusions

The regulator formulated in this way is a new contribution in the area of MASV method theory. In the following articles we will deal with problems mentioned above and with modelling of a regulator in PC program MATLAB, in concrete applications.

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